The Equivalence of Two-Dimensional PCA to Line-Based PCA

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Abstract

The state-of-the-art in human face recognition is the subspace methods originated by the Principal Component Analysis (PCA), the Eigenfaces of the facial images. Recently, a technique called Two-Dimensional PCA (2DPCA) was proposed for human face representation and recognition. It was developed for image feature extraction based on 2D matrices as opposed to the standard PCA, which is based on 1D vectors. In this note, we show that 2DPCA is equivalent to a special case of an existing feature extraction method, block-based PCA, which has been used for face recognition in a number of systems.

Keywords: PCA; Human face recognition; Two-Dimensional PCA; Block-based methods

1. Introduction

Human face recognition has become an active area of research over the last decade. A pioneer work is the Karhunen-Loeve transform of faces (Kirby and...

PCA (Eigenface method) considers images as vectors in a high dimensional image space. All the facial images are projected onto the eigenspace spanned by the leading vectors of the sample covariance matrix of the training images. The final identification is performed with a nearest neighbor classifier. PCA has become a \textit{de facto} standard and a common performance benchmark in the field. The state-of-the-art in face recognition is characterized by a family of subspace methods. Some representatives are Fisherfaces (PCA+LDA) (Belhumeur et al., 1997; Etemad and Chellappa, 1997), Bayesian similarity (Moghaddam et al., 2000), ICA (Hyvarinen et al., 2001; Bell and Sejnowski, 1995; Comon, 1994), and their nonlinear generalizations using the kernel trick (Scholkopf et al., 1998; Mika et al., 1999). These algorithms have demonstrated a good performance in various datasets.

More recently, a technique called Two-dimensional PCA (2DPCA) (Yang and Zhang, 2004) (also referred to as image PCA (IMPCA) in a previous paper by Yang and Yang (2002)) was proposed to cut the computational cost of the standard PCA. Unlike PCA that treats images as vectors, 2DPCA views an image as a matrix. With a proper criterion, 2DPCA results in an eigenvalue problem, but has a much lower dimensionality than PCA.

Another line of research in feature extraction and face recognition is the block-based methods. In these methods, an image is partitioned into several blocks. Often, all the blocks have same size. Features are extracted from the block images.
Block-based method first appeared in Hidden Markov Models (HMM) based face recognition (Kim et al., 2003) dating back to Samaria and Young’s paper (1994). Other works related to this method include fragment-based feature extraction (Vidal-Naquet et al., 2003; Ullman et al., 2001), which makes use of image fragments to represent objects.

In this note, we show that 2DPCA (IMPCA) is equivalent to a special case of the block image based PCA. When each block is a line of the image, and taking all the lines of the facial images as training samples, applying standard PCA to these “line” samples obtains the 2DPCA. We also compare 2DPCA to the typical rectangle block (e.g. 8×8 window) based PCA in a face recognition task. The experimental results demonstrate that 2DPCA achieves higher recognition rate than rectangle block PCA.

The rest of this paper is organized as follows. Section 2 briefly reviews the 2DPCA. Section 3 addresses the block-based PCA, and we show the equivalence of 2DPCA to line-based PCA in Section 4. In Section 5, we give the experimental results.

2. Two-Dimensional PCA (2DPCA)

Consider an $M$ by $N$ image as $M \times N$ random matrix denoted by $A$. Let $x$ be an $N$-dimensional unit column vector. Projecting $A$ onto $x$ yields an $M$-dimensional vector $y$.

$$ y = Ax. \quad (1) $$

The purpose of 2DPCA is to select a good projection vector $x$. To evaluate the
goodness of a projection vector, the authors suggested the use of the total scatter of the projected samples, which can be characterized by the trace of the covariance matrix of the projected feature vectors. Thus, the criterion is to maximize the following:

\[ J(x) = \text{tr}(S_x). \]  

(2)

where \( S_x \) is the covariance matrix of the projected feature vectors, written by

\[ S_x = E(y - Ey)(y - Ey)^T = E[(A - EA)x][(A - EA)x]^T. \]  

(3)

Hence,

\[ J(x) = \text{tr}(S_x) = x^T E[(A - EA)(A - EA)^T]x. \]  

(4)

Given a set of training images \( A(1), A(2), \ldots, A(n) \), the criterion (4) becomes

\[ J(x) = x^T \left[ \frac{1}{n} \sum_{i=1}^{n} (A(i) - \bar{A})^T (A(i) - \bar{A}) \right] x. \]  

(5)

where \( \bar{A} \) is the average of all training images. For simplicity, we will drop the normalizing constant \( 1/n \) of the covariance matrix in the rest of this paper. Let

\[ G = \sum_{i=1}^{n} (A(i) - \bar{A})^T (A(i) - \bar{A}), \]

the optimal axis \( x_{opt} \) is the unit vector maximizing \( J(x) \), i.e. the eigenvector of \( G \) corresponding to the largest eigenvalue. Of course, one can compute \( m \) best projection axes, which are the \( m \) leading eigenvectors of \( G \).

Without loss of generality, we will always assume that all the images have been shifted so that they have zero mean, i.e. \( \bar{A} = \frac{1}{n} \sum_{i=1}^{n} A(i) = (0)_{M \times N} \). Thus, (5) becomes

\[ J(x) = x^T \left[ \sum_{i=1}^{n} A(i)^T A(i) \right] x. \]  

(6)
3. Block-Based PCA

Let \( A(1), A(2), \cdots, A(n) \) be the training images. Block image based PCA is to first divide each image into several blocks. Usually, all the blocks have same size. For example, it can be an \( 8 \times 8 \) window, or a few lines. One method of performing block-based PCA is to view each block as a sample image. If one divides an image into \( r \) blocks, there are totally \( r \times n \) training samples. Computing the leading vectors of the sample covariance matrix of these \( r \times n \) blocks obtains the block image principal components.

4. The Equivalence of 2DPCA to Line-Based PCA

In this section, we show that 2DPCA is equivalent to a special case of the block image based PCA. Specifically, the blocks are the lines of the images, i.e. each block is a line of the raw image.

Suppose again, that \( A(1), A(2), \cdots, A(n) \) are training images of size \( M \times N \). Let \( a(i)_1, a(i)_2, \cdots, a(i)_M \) be the \( M \) lines of \( A(i) \), \( i = 1, 2, \cdots, n \). More concretely,

\[
A(i) = \begin{bmatrix}
a(i)_{11} & a(i)_{12} & \cdots & a(i)_{1N} \\
a(i)_{21} & a(i)_{22} & \cdots & a(i)_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
a(i)_{M1} & a(i)_{M2} & \cdots & a(i)_{MN}
\end{bmatrix}
= \begin{bmatrix}
a(i)^T_1 \\
a(i)^T_2 \\
\vdots \\
a(i)^T_M
\end{bmatrix}, \quad i = 1, 2, \cdots, n \tag{7}
\]

and

\[
a(i)^T_m = (a(i)_m^1, a(i)_m^2, \cdots, a(i)_m^N), \quad i = 1, 2, \cdots, n, \quad m = 1, 2, \cdots, M. \tag{8}
\]

Consider the criterion of 2DPCA (see (6)). Let \( G = \sum_{i=1}^n A(i)^T A(i) \), 2DPCA is to compute the leading eigenvectors of \( G \). Taking into consideration of (7), (8), and
rewriting $G$ in terms of $a(i)_m$, $i = 1, 2, \cdots, n$, $m = 1, 2, \cdots, M$, we have

$$G = \sum_{i=1}^{n} A(i)^T A(i) = \sum_{i=1}^{n} \sum_{m=1}^{M} a(i)_m a(i)_m^T. \quad (9)$$

Clearly, $G$ is the sample covariance matrix of all the lines $a(i)_m$, $i = 1, 2, \cdots, n$, $m = 1, 2, \cdots, M$. That is, if we consider each line of each training image as a sample vector, then $G$ is the sample covariance matrix. 2DPCA is therefore equivalent to the line-based PCA, which is a special case of the block image based PCA.

5. Experiments

The primary goal of the experiments in this section is to compare 2DPCA (i.e. line based PCA) to the typical rectangle block (e.g. $8 \times 8$ window) based PCA. The comparison is conducted through a human face recognition task. We use the recognition accuracy to show the relative strength and weakness of the two feature extraction methods.

The facial images in the experiments are all from the well-known FERET database. For details about this dataset and the terminologies used below, we refer the reader to (Phillips et al. 2000). The gallery contains 1196 images, and the probe (so-called FB probe) consists of 1195 images. We select 1068 frontal images from 554 individuals as the training set, from which the two types of features are extracted.

Recognition is similar to the Eigenfaces method. Please see (Yang and Zhang, 2004) for detail. The performance statistics is cumulative match score. We plot the recognition result in Fig. 1 (up to the top 50 candidates).

The experimental results suggest that 2DPCA (line block) is better than
Fig. 1. Recognition rates of 2DPCA (line block) and rectangle block PCA on FERET dataset.

In this note, we show that 2DPCA (also referred to as IMPCA) is a special case of the block image based PCA. Specifically, the blocks are the lines of the raw images. We also demonstrated that 2DPCA achieves higher accuracy than the typical rectangle block based PCA on the FERET FB probe.

6. Conclusion

In this note, we show that 2DPCA (also referred to as IMPCA) is a special case of the block image based PCA. Specifically, the blocks are the lines of the raw images. We also demonstrated that 2DPCA achieves higher accuracy than the typical rectangle block based PCA on the FERET FB probe.

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Reference


